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DUST-ION-ACOUSTIC PRECURSOR OF A SHOCK WAVE

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The effect of charged dust particles on the structure of the plasma precursor of a strong shock wave is studied. The conditions of formation of a weak discontinuity front are obtained. It is shown that resonant modes can occur in which the concentration of dust particles in the neighborhood of the front increases. In the case of positively charged particles of dust, the formation of a localized compaction region in the form of a soliton "bunch" is possible and the dependence of the amplitude of the soliton on shock-wave velocity is nonmonotonic. In the case of negatively charged particles of dust, a rarefaction wave is formed. The indicated phenomena can substantially affect the concentration of the neutral component in a slightly ionized plasma.

Introduction. Recently, interest in nonlinear processes in partly and slightly ionized, multicomponent plasmas has considerably increased [1–9]. This is motivated by studies of the structure of planetary rings and cometary tails, flows of the solar wind around the Earth, the formation of stars, the nature of ball lightning, the formation of plasma-dust crystals, the evolution and formation of shock-wave structures in dust plasmas, abnormal influence of the plasma component on the flow of a slightly ionized plasma around bodies, the structure of the shock-wave precursor in a plasma, etc. The multicomponent nature of plasmas can be ensured by the presence of various sorts of ions and charged macroscopic particles (dust, aerosols, clusters, etc.). Usually, a dust-plasma system is open and exists in the presence of an extraneous source. In this case, the charged particles of dust can be approximately treated as an additional heavy plasma component for which the condition of quasineutrality relative to the dust is satisfied [6]:

$$(4/3)\pi n_{\rm d} D_{\rm d}^3 \gg 1 \tag{1}$$

or

$$n_{\rm d}^{1/2} Z_{\rm d}^3 \ll (4 \cdot 10^8) T_{\rm d}^{3/2}.$$

Here $n_{\rm d}$ [cm^{-3}] is the concentration, $D_{\rm d}$ is the Debye radius, $T_{\rm d}$ [eV] is the temperature, and $Z_{\rm d}$ is the charging number (charge of a dust particle in electron charge units); the subscript "d" corresponds to dust particles.

Inequality (1) corresponds to the state of a dust system with a "small" concentration and a "small" charge. If condition (1) is violated, the dust component becomes "special" [6]: it is not described by the approximation of a continuous medium, and, hence, kinetic effects should be taken into account in this case. This situation is encountered in problems of the formation of dust-plasma crystals, drops, and clouds. There are a number of phenomena related to the electrification of dust particles [5].

Usually, an unperturbed plasma is electrically neutral:

$$n_{\rm i0} = n_{\rm e0} + Z_{\rm d} n_{\rm d0}.$$

Here and below, the subscripts "i" and "e" and correspond to ions and electrons; the subscript 0 characterizes the unperturbed state; $Z_{\rm d} > 0$ for negatively charged dust particles, and $Z_{\rm d} < 0$ for positively charged particles.

In some cases, an unperturbed plasma is slightly ionized (see, e.g., [6, 10–12]): $\delta_0 \equiv n_{\rm i0} n_{\rm n0}^{-1} = 10^{-5}$–$10^{-7}$ (the subscript "n" corresponds to neutral particles). This simplifies the description of the interaction of a shock

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wave with the plasma components in the initial stage of formation of regions with elevated charge concentration; in this case, the action of charges on the neutral component is ignored. At this stage, the field of the neutral component is considered specified: it is a source of perturbations in the plasma. For flows of a slightly ionized dust plasma around bodies under laboratory conditions, condition (1) is satisfied in most cases and $Z_{\rm d} \approx \text{const}$ [11, 12].

The joint action of strong nonlinearity and dispersion can result in the formation of localized regions of elevated ionization level [12–14]. In this case, the effect of charges on neutral particles cannot be ignored. Elastic collisions and other interaction mechanisms in these regions can lead to a strong effect of charged particles on the neutral component. In addition, in these region, violation of condition (1) is possible: dust begins to "scatter" the shock wave due to the flow of "hot" neutral particles ($T_{\rm n} > T_{\rm d}$) on the heavy dust particles. Under these conditions there may be significant attenuation of the shock-wave intensity and even failure of the shock wave.

1. The formulation of the problem is similar to that in [13, 14]. A new point is consideration of the additional charged dust component, for which the quasineutrality condition (1) is satisfied. We study a one-dimensional steady perturbation in a slightly ionized, nonisothermal plasma ($T_{\rm e} \gg T_{\rm i} \approx T_{\rm n} > T_{\rm d}$) produced by a strong shock wave of the neutral component. This shock wave is specified as

$$V_{\rm n} = V_{\rm n1}\eta(-\xi), \quad \xi = x - ct, \quad \eta(x) = \begin{cases} 1, & x > 0, \\ 0, & x < 0, \end{cases} \qquad \rho_{\rm n} = \rho_{\rm n0} + (\rho_{\rm n1} - \rho_{\rm n0})\eta(-\xi),$$

where x and t are the coordinate and time, $c = \text{const}$ is the shock-wave speed, and $V_{\rm n}$ and $\rho_{\rm n}$ are the velocity and density of the neutral component; the subscripts 0 and 1 correspond to the states ahead of and behind the front.

Taking into account the slight ionization of the plasma, we ignore the effect of charged particles on the neutral component. The processes in the plasma are described by the following system of equations [condition (1) is assumed to be satisfied]:

$$\begin{gathered}
\frac{\partial n_j}{\partial t} + \frac{\partial}{\partial x}(n_j V_j) = 0, \qquad j = \{\rm i, e, d\}, \\
\Big(\frac{\partial}{\partial t} + V_{\rm i}\frac{\partial}{\partial x}\Big)V_{\rm i} = |e| m_{\rm i}^{-1} E - \nu_{\rm i}(V_{\rm i} - V_{\rm n}), \\
0 = -n_{\rm e}|e|E - kT_{\rm e}\frac{\partial n_{\rm e}}{\partial x}, \\
\Big(\frac{\partial}{\partial t} + V_{\rm d}\frac{\partial}{\partial x}\Big)V_{\rm d} = -Z_{\rm d}|e| m_{\rm d}^{-1} E - \nu_{\rm d}(V_{\rm d} - V_{\rm n}), \\
\varepsilon_0\frac{\partial E}{\partial x} = |e|(n_{\rm i} - n_{\rm e} - Z_{\rm d} n_{\rm d}), \qquad n_{\rm i0} - n_{\rm e0} - Z_{\rm d} n_{\rm d0} = 0.
\end{gathered} \tag{2}$$

Here $T_{\rm e} \gg T_{\rm i}$, $T_{\rm e} \gg T_d$, $T_{\rm e} = \text{const}$, $Z_{\rm d} = \text{const}$, V_j, and T_j are the velocity and temperature of the corresponding plasma component, m is the mass, $-|e|$ is the electron charge, E is the electric field, ε_0 is the permittivity of free space, and k is the Boltzmann constant.

For $Z_{\rm d} = 0$, system (2) describes the ion-acoustic waves produced by the extraneous source ($V_{\rm n}$). The situation $Z_{\rm d} \neq 0$ corresponds to the case of charged particles of dust. The fields described by system (2) will be referred to as dust-ion-acoustic fields (this term is not conventional). Because the process is steady-state (the parameter fields depend on the single variable ξ), the partial equations reduce to the following system of three coupled ordinary differential equations for the functions $V_{\rm i}$, $n_{\rm d}$, and Ψ:

$$\frac{d}{d\xi}\Big[\frac{1}{2}\Big(\frac{V_{\rm i} - c}{c}\Big)^2 + \frac{\Psi}{{\rm M}_{\rm i}^2}\Big] = -\frac{\nu_{\rm i}}{c^2}(V_{\rm i} - V_{\rm n}); \tag{3}$$

$$\frac{d}{d\xi}\Big[\frac{1}{2}\Big(\frac{n_{\rm d0}}{n_{\rm d}}\Big)^2 - \frac{\beta\Psi}{{\rm M}_{\rm i}^2}\Big] = -\frac{\nu_{\rm d}}{c}\Big(1 - \frac{n_{\rm d0}}{n_{\rm d}} - \frac{V_{\rm n}}{c}\Big); \tag{4}$$

$$V_{\rm i}/c = 1 - (1 + \alpha)/(F(\Psi) + \alpha n_{\rm d} n_{\rm d0}^{-1}). \tag{5}$$

Here $F(\Psi) = \exp\Psi - 2D_{\rm e}^2\,(d^2\Psi/d\xi^2)$, $D_{\rm e}^2 = \varepsilon_0 kT_{\rm e}/(2n_{\rm e0}e^2)$, $V_s^2 = kT_{\rm e}/m_{\rm i}$, $\alpha = Z_{\rm d} n_{\rm d0}/n_{\rm e0}$, $\beta = Z_{\rm d} m_{\rm i}/m_{\rm d}$, and ${\rm M}_{\rm i} = c/V_s$ is the ion Mach number. In deriving Eq. (5), we used boundary conditions that assume the existence of an unperturbed state in the laboratory coordinate system:

$$\xi = \infty: \qquad V_{\rm i} = 0, \quad n_{\rm d} = n_{\rm d0}, \quad n_{\rm i} = n_{\rm i0}, \quad \Psi = 0.$$

The fields of the parameters E, $n_{\rm e}$, $n_{\rm i}$, and $V_{\rm d}$ are related to the solutions of system (3)–(5) by formulas

$$E = -m_{\rm i} V_s^2 |e|^{-1} \frac{d\Psi}{d\xi}, \quad n_{\rm e} = n_{\rm e0} \exp \Psi, \quad n_{\rm i}(V_{\rm i} - c) = -n_{\rm i0} c, \quad n_{\rm d}(V_{\rm d} - c) = -n_{\rm d0} c.$$

At infinity, the unperturbed state takes the form

$$E_j(\infty) = 0, \qquad n_j(\infty) = n_{j0}, \quad j = \{\rm i, e, d\}.$$

We first consider the limiting case (long-wave approximation) $D_{\rm e} = 0$, which corresponds to the description of slowly varying fields ignoring higher derivatives. In the neighborhood of the fronts there are high field gradients and the role of higher derivatives is important; therefore, the following remark is relevant.

2. For $Z_{\rm d} = 0$ (no charged particles of dust) in the long-wave approximation ($D_{\rm e} = 0$) the gradients of the parameters in the precursor increase with increase in the ionic Mach number $\mathrm{M}_{\rm i}$ to unity. For $\mathrm{M}_{\rm i} = 1$, the first derivatives undergo a discontinuity (weak) at a certain point $\xi = \xi_*$ [14], which is due to the absence of a continuous solution satisfying the boundary condition at $\xi = \infty$. Let us show that in the presence of dust, a weak discontinuity can also exist. This condition holds in the long-wave approximation for $\mathrm{M}_{\rm i} = \mathrm{M}_h \equiv \sqrt{1 + \alpha(1 + \beta)}$. We assume that such a discontinuity exists and its coordinate is $\xi = \xi_h$. Then, for $\xi > \xi_h$, the state of the fields should be unperturbed, and for $\xi < \xi_h$ (behind the front) in the neighborhood $\xi \approx \xi_h$, the fields can be written as expansions

$$V_{\rm i} = V_h'(\xi - \xi_h) + 0{,}5 V_h''(\xi - \xi_h)^2 + \ldots,$$

$$\Psi = \Psi_h'(\xi - \xi_h) + 0{,}5\Psi_h''(\xi - \xi_h)^2 + \ldots, \qquad n_{\rm d} = n_{\rm d0} + n_h'(\xi - \xi_h) + 0{,}5 n_h''(\xi - \xi_h)^2 + \ldots,$$

where the fields denoted by the subscript h are constant quantities; primes denote derivatives. Using these expansions for zeroth-order terms in (3) and (4) and for first-order terms in (5), we obtain the following closed system of equations in the region $\xi < \xi_h$:

$$c^{-1} V_h' - \mathrm{M}_h^{-2} \Psi' = 0, \quad n_{\rm d0}^{-1} n_h' + \beta \mathrm{M}_h^{-2} \Psi_h' = 0, \quad \Psi_h' + \alpha n_{\rm d0}^{-1} n_h' - (1 + \alpha) V_h' = 0. \tag{6}$$

From the condition of resolvability of the homogeneous system (6) it follows that

$$\mathrm{M}_h^2 = 1 + \alpha(1 + \beta).$$

[Below it is shown that the value of M_h coincides with the representation for the lower boundary $\mathrm{M}_{\rm i}$ of existence of steady-state solutions for a collisionless plasma ($\nu_{\rm i} = \nu_{\rm d} = 0$ and $D_{\rm e} \neq 0$).]

The first-order terms from (3) and (4) and the second-order terms from (5) form the system

$$\begin{gathered} -c^{-2}[c V_h'' - (V_h')^2] + \mathrm{M}_h^{-2} \Psi_h'' = -c^{-2} \nu_{\rm i} V_h', \quad -3 n_{\rm d0}^{-2} (n_h')^2 + n_{\rm d0}^{-1} n_h'' + \beta \mathrm{M}_h^{-2} \Psi_h'' = c^{-1} \nu_{\rm d} n_{\rm d0}^{-1} n_h', \\ \Psi_h'' + (\Psi_h')^2 + \alpha n_{\rm d0}^{-1} n_h'' - c^{-1}(1 + \alpha) V_h'' - 2(1 + \alpha) c^{-2} (V_h')^2 = 0. \end{gathered} \tag{7}$$

From Eqs. (6) and (7) we obtain the expressions for the first derivatives of the fields behind the weak-discontinuity front at $\xi < \xi_h$:

$$\begin{gathered} V_h' = -[\nu_{\rm i}(1 + \alpha) + \alpha\beta\nu_{\rm d}][(1 + \beta)(\alpha - \alpha_1(\beta))(\alpha - \alpha_2(\beta))]^{-1}, \quad \Psi_h' = c^{-1} \mathrm{M}_h^2 V_h'; \\ n_h' = -\beta n_{\rm d0} c^{-1} V_h'. \end{gathered} \tag{8}$$

Here $\alpha_{1,2} \equiv [2(1 + \beta)]^{-1}[1 - 2\beta - 3\beta^2 \pm \sqrt{(1 - 2\beta - 3\beta^2)^2 + 8(1 - \beta^2)}\,]$. As was noted above, ahead of the weak-discontinuity front ($\xi > \xi_h$) there is an unperturbed state $V_{\rm i} = 0$, $\Psi = 0$, and $n_{\rm d} = n_{\rm d0}$.

In the absence of dust particles ($Z_{\rm d} = 0$), we obtain the well-known result [14]: $\mathrm{M}_h = 1$ and $V_h' = -\nu_{\rm i}/2$. For positively charged dust, $\mathrm{M}_h < 1$ at $-1 < \beta < 0$. If $\beta < -1$, then $\mathrm{M}_h > 1$. It should be noted that in most situations that arise, $|\beta| \ll 1$ but there may be conditions under which $|\beta| > 1$. For $\alpha = -1$, a rare situation arises: as a first approximation, ion-neutral collisions do not influence the structure of the field in the neighborhood of the front (the parameters V_h', Ψ_h', and n_h' do not depend on $\nu_{\rm i}$). According to (8), the derivatives V_h' and n_h' for $\beta < 0$ have the same sign, and for negatively charged dust, $\mathrm{M}_h > 1$ and $\operatorname{sign} V_h' = -\operatorname{sign} n_h'$. The last relation implies the existence of a rarefaction wave in the dust component in the case of negatively charged dust: $n_{\rm d} < n_{\rm d0}$. An interesting circumstance is the presence of "resonant" properties of a dust plasma. In the present formulation of the problem considered, in the regime $\mathrm{M}_{\rm i} \to \mathrm{M}_h$, the parameters V_h', Ψ_h', and n_h' can become infinite as $\alpha \to \alpha_{1,2}(\beta)$.

The production of a dust plasma with parameters satisfying the relation $\alpha = \alpha_{1,2}(\beta)$ is of special interest because in the regime $\mathrm{M}_{\mathrm{i}} \to \mathrm{M}_h$, a region of elevated concentration of dust particles should form.

3. We study the structure of small-amplitude fields for $\mathrm{M}_{\mathrm{i}} \neq \mathrm{M}_h$ away from the front $\xi = 0$, restricting ourselves to a linear approximation of Eqs. (3)–(5):

$$c^{-1}\frac{dV_{\mathrm{i}}}{d\xi} - \mathrm{M}_{\mathrm{i}}^{-2}\frac{d\Psi}{d\xi} = \nu_{\mathrm{i}}c^{-2}V_{\mathrm{i}}, \quad n_{\mathrm{d0}}^{-1}\frac{dn_{\mathrm{d}}}{d\xi} + \beta\mathrm{M}_{\mathrm{i}}^{-2}\frac{d\Psi}{d\xi} = \nu_{\mathrm{d}}c^{-1}n_{\mathrm{d0}}^{-1}(n_{\mathrm{d}} - n_{\mathrm{d0}}),$$

$$\Psi = -\alpha n_{\mathrm{d0}}^{-1}(n_{\mathrm{d}} - n_{\mathrm{d0}}) + (1+\alpha)c^{-1}V_{\mathrm{i}}.$$

Let us determine under what conditions losses lead to an exponential decrease of the fields in the region $\xi \to \infty$:

$$V_{\mathrm{i}} \sim \Psi \sim (n_{\mathrm{d}} - n_{\mathrm{d0}}) \sim \exp(-\xi/\xi_0), \qquad \xi_0 > 0$$

[ξ_0 satisfies the equation $\xi_0^2 A_1 + \xi_0 A_2 + A_3 = 0$, $A_1 = \nu_{\mathrm{i}}\nu_{\mathrm{d}}\mathrm{M}_{\mathrm{i}}^2 c^{-2}$, $A_2 = \nu_{\mathrm{i}}c^{-1}(\mathrm{M}_{\mathrm{i}}^2 - \alpha\beta) + \nu_{\mathrm{d}}c^{-1}(\mathrm{M}_{\mathrm{i}}^2 - \mathrm{M}_h^2 + \alpha\beta)$, and $A_3 = \mathrm{M}_{\mathrm{i}}^2 - \mathrm{M}_h^2$].

The coefficients A_n have the following properties: $A_1 > 0$, $A_{2,3} > 0$ for $\mathrm{M}_{\mathrm{i}} > \mathrm{M}_h$ and $A_3 < 0$ for $\mathrm{M}_{\mathrm{i}} < \mathrm{M}_h$. When the condition $\mathrm{M}_{\mathrm{i}} < \mathrm{M}_h$ is satisfied, the parameter ξ_0 is positive, i.e. away from the shock-wave front, the precursor enters the unperturbed state under an exponential law. If $\mathrm{M}_{\mathrm{i}} > \mathrm{M}_h$, the parameter ξ_0 is negative, i.e., there is no continuous precursor that enters the unperturbed state as $\xi \to +\infty$. For $\mathrm{M}_{\mathrm{i}} > \mathrm{M}_h$, the continuous solution of system (3)–(5) becomes ambiguous (cf. the case of $Z_{\mathrm{d}} = 0$ in [14]) and such that the unperturbed state exists for $\xi = +\infty$ rather than for $\xi = -\infty$: $V_{\mathrm{i}}(-\infty) = 0$, $\Psi(-\infty) = 0$, and $n_{\mathrm{d}}(-\infty) = n_{\mathrm{d0}}$. To satisfy the boundary conditions at infinity $\xi = +\infty$, it is necessary to introduce a discontinuity.

4. Before introducing a discontinuity for $\mathrm{M}_{\mathrm{i}} > \mathrm{M}_h$, we study the properties of the continuous ambiguous solution of system (3)–(5) for $\mathrm{M}_{\mathrm{i}} > \mathrm{M}_h$ in the neighborhood of the point $\xi = \xi_m$, where the first derivatives of the fields become infinite. With satisfaction of the condition $\xi \leqslant \xi_m$, the fields can be expanded as

$$V_{\mathrm{i}} = V_m + c\sum_{n=0}^{\infty}(\xi_m - \xi)^{(n+1)/2} f_{\mathrm{n}},$$

$$n_{\mathrm{d}} = n_m + n_{\mathrm{d0}}\sum_{n=0}^{\infty}(\xi_m - \xi)^{(n+1)/2}\varphi_{\mathrm{n}}, \qquad \Psi = \Psi_m + \sum_{n=0}^{\infty}(\xi_m - \xi)^{(n+1)/2}\Phi_{\mathrm{n}}.$$

The main terms of the expansion have the form

$$f_0 = \pm\sqrt{2\nu_{\mathrm{i}}V_m c^{-2}},$$

$$\varphi_0 = \pm n_m n_{\mathrm{d0}}\sqrt{2\nu_{\mathrm{d}}(3c)^{-1}(1 - n_{\mathrm{d0}}n_m^{-1})}, \qquad \Phi_0 = \pm\mathrm{M}_{\mathrm{i}}^2(1 - V_m c^{-1})\sqrt{2\nu_{\mathrm{i}}V_m c^{-2}},$$

and the dimensionless parameters $y = n_m/n_{\mathrm{d0}}$ and $z = 1 - V_m/c$ satisfy the nonlinear equations

$$\frac{\sqrt{y(y-1)}}{z\sqrt{1-z}} = -\beta\sqrt{3\nu_{\mathrm{i}}\nu_{\mathrm{d}}^{-1}}, \qquad \frac{\beta + \mathrm{M}_{\mathrm{i}}^2 y}{y} = \frac{(1+\alpha)(1 - \mathrm{M}_{\mathrm{i}}^2 z^2)}{\alpha z^3},$$

and Ψ_m is obtained from the relation

$$\exp\Psi_m = -\alpha n_m n_{\mathrm{d0}}^{-1} + (1+\alpha)(1 - V_m c^{-1})^{-1}.$$

In the particular case of no dust particles, we obtain the well-known result [14]: $1 - V_m c^{-1} = \mathrm{M}_{\mathrm{i}}^{-1}$ and $\Psi_m = \mathrm{M}_{\mathrm{i}}$.

In the case of positively charged dust, the coefficients f_0, φ_0, and Φ_0 are real, and the expression for the shock-wave precursor is continuous and ambiguous. By a standard procedure (see, e.g., [14]), this expression is transformed to a discontinuous solution that describes a dust-ion-acoustic shock wave. For this, we employ the relations for the discontinuity that follow from the laws of conservation of mass and momentum of ions and dust in integral form:

$$-u[n_j] + [n_j V_j] = 0, \qquad j = \{\mathrm{i}, \mathrm{e}, \mathrm{d}\},$$

$$[V_s^2 n_{\mathrm{i}}V_{\mathrm{i}} + n_{\mathrm{i}}V_{\mathrm{i}}(V_{\mathrm{i}} - u)] = 0, \qquad [-\beta V_s^2 n_{\mathrm{d}} + n_{\mathrm{d}}V_{\mathrm{d}}(V_{\mathrm{d}} - u)] = 0.$$

Here u is the rate of displacement of the discontinuity; square brackets denote a discontinuity of the corresponding function at the dust-ion-acoustic wave front.

Because of the steadiness of the problem, $u = c$. Unlike in problems of the dynamics of a continuous neutral medium, in the formulation considered the discontinuity field is formed in a dissipative medium: $\nu_{\rm i} \neq 0$, $\nu_{\rm d} \neq 0$, and $D_{\rm e} = 0$.

In the case of negatively charged dust, the coefficients f_0, φ_0, and Φ_0 become purely imaginary because $y > 1$. This indicates that negatively charged dust prevents the formation of a discontinuity field and serves as indirect evidence for disturbance of the steadiness. Therefore, it is necessary to refine the model that describes the negatively charged dust plasma in the regime $\mathrm{M}_{\rm i} \geqslant \mathrm{M}_h$ (allowance for multiflow motion, turbulence, viscosity, etc.).

For positively charged dust in the neighborhood of a dust-ion-acoustic front, the higher derivatives in system (3)–(5) become essential and the role of dispersion ($D_{\rm e} \neq 0$) is significant. For $\mathrm{M}_{\rm i} > \mathrm{M}_h$, a negatively charged dust plasma cannot be described using model (2) because a real solution of the problem is absent.

5. We study the steady-state solutions of system (3)–(5) in the limiting case of a collisionless plasma ($\nu_{\rm i} = \nu_{\rm d} = 0$). A similar problem was considered in [7–9]. System (3)–(5) reduces to the following second-order equation for the electric-field potential:

$$2D_{\rm e}^2\Psi'' = \exp\Psi - n_{\rm i0}n_{\rm e0}^{-1}(1 - 2\mathrm{M}_{\rm i}^{-2}\Psi)^{-1/2} + \alpha(1 + 2\beta\mathrm{M}_{\rm i}^{-2}\Psi)^{-1/2}. \qquad (9)$$

Equation (9) is an extension of the Sagdeev equation [15, Eq. (78)] to the case of allowance for the effect of the third charged component for arbitrary values of the dimensionless parameters α and β. Integrating (9), we obtain the first-order equation

$$D\Psi' = \pm\sqrt{\Phi^{(0)} - \Phi(\Psi)}, \qquad \Phi^{(0)} = \mathrm{const},$$
$$\Phi(\Psi) = 1 - \exp\Psi + \mathrm{M}_{\rm i}^2(1+\alpha)\Big(1 - \sqrt{1 - 2\mathrm{M}_{\rm i}^{-2}\Psi}\Big) + \mathrm{M}_{\rm i}^2\alpha\beta^{-1}\Big(1 - \sqrt{1 + 2\beta\mathrm{M}_{\rm i}^{-2}\Psi}\Big), \quad \Phi(0) = 0. \qquad (10)$$

The steady-state waves described by Eqs. (9) and (10) exist provided that the function $\Phi(\Psi)$ has a minimum rather than a maximum: $\Phi'(\Psi) < 0$, $\Psi \ll 1$, and $\Psi > 0$. From this we have the lower bound for the ion Mach number

$$\mathrm{M}_{\rm i} > \sqrt{1 + \alpha(1+\beta)} \equiv \mathrm{M}_h.$$

(The representation $\mathrm{M}_{\rm i}$, which ensures the creation of a weak discontinuity for $D_{\rm e} = 0$, $\nu_{\rm i} \neq 0$, and $\nu_{\rm d} \neq 0$ is given in Sec. 2.)

In the limiting case of ion-acoustic waves, we obtain the well-known condition $\mathrm{M}_{\rm i} > 1$. The value $\mathrm{M}_h = 0$ gives the upper bound for the concentration of positive dust particles $|\alpha|(1 - |\beta|) > 1$. If the last inequality is violated, M_h becomes an imaginary quantity and, hence, refinement of the calculation model is required. The fields E, n_j, and V_j are related to Ψ by the formulas

$$E = -m_{\rm i}V_s^2|e|^{-1}\Psi', \qquad n_{\rm e}n_{\rm e0}^{-1} = \exp\Psi,$$

$$n_{\rm i}n_{\rm i0}^{-1} = c(c - V_{\rm i})^{-1} = (1 - 2\mathrm{M}_{\rm i}^{-2}\Psi)^{-1/2}, \qquad n_{\rm d}n_{\rm d0}^{-1} = c(c - V_{\rm d})^{-1} = (1 + 2\mathrm{M}_{\rm i}^{-2}\Psi\beta)^{-1/2}.$$

For $\beta > 0$, the concentration of dust particles decreases: $n_{\rm d} < n_{\rm d0}$ (a rarefaction wave arises for negatively charged dust), and for $\beta < 0$ and $n_{\rm d} > n_{\rm d0}$ (a compression wave arises for positively charged dust).

Because physically realistic fields are described by real functions, it follows that in the present formulation of the problem, the following conditions should be satisfied:

$$\Psi \leqslant \Psi_* \equiv 2^{-1}\mathrm{M}_{\rm i}^2, \qquad \beta > -1$$

or

$$\Psi \leqslant \Psi_{**} \equiv -2^{-1}\beta^{-1}\mathrm{M}_{\rm i}^2, \qquad \beta < -1.$$

The solutions of Eq. (10) have properties typical of solutions of the Sagdeev equations [15]:

— for $\Phi^{(0)} < 0$, only periodic solutions exist;

— for $\Phi^{(0)} = 0$, solutions in the form of a solitary wave (soliton) exist;

— a soliton with maximum amplitude exists.

In the presence of dust particles there may be two representations for the maximum amplitude of the soliton: Ψ_* or Ψ_{**} ($\Psi_* = 2^{-1}\mathrm{M}_*^2$ for $\beta > -1$ and $\Psi_{**} = -2^{-1}|\beta|^{-1}\mathrm{M}_{**}^2$ for $\beta < -1$).

For $\beta > -1$, a soliton of maximum amplitude Ψ_* is described by the equation $\Phi(\Psi_*) = 0$, which reduces to the form

$$\exp(2^{-1}\mathrm{M}_*^2) = 1 + A\mathrm{M}_*^2,$$

where $A(\alpha, \beta) = 1 + \alpha + \alpha\beta^{-1}(1 - \sqrt{1+\beta})$.

For $\beta < -1$, a soliton of maximum amplitude Ψ_{**} is described by the equation $\Phi(\Psi_{**}) = 0$, which reduces to the form

$$\exp(2^{-1}|\beta|^{-1}\mathrm{M}_{**}) = 1 + B\mathrm{M}_{**},$$

where $B(\alpha, \beta) = (1 - |\alpha|)(1 - \sqrt{1 - |\beta|^{-1}}) + \alpha\beta^{-1}$.

Generally, the critical values M_*, M_{**}, Ψ_*, and Ψ_{**} depend on both the relative concentration (parameter α) and on the mass ratio (parameter β). In the particular case of $|\beta| \ll 1$, we have $\max\Psi \approx \Psi_*(\alpha)$.

6. Let us consider the main properties of the dust-ion -acoustic precursor of a shock wave. In the situations described in Secs. 2 and 4, weak and strong discontinuities can form, the role of higher derivatives in system (3)–(5) is important, and the effect of spatial dispersion is therefore should be taken into account. A soliton plasma "bunch" forms at the leading edge (cf. the case $Z_\mathrm{d} = 0$ in [14]). The occurrence of this soliton formation is due to the smoothing of the shock due to the simultaneous action of dispersion and nonlinearity. In the case of positively charged dust, the amplitude of the soliton increases as the ion Mach number increases from M_h to M_* for $\beta > -1$ and to $\mathrm{M}_\mathrm{i} = \mathrm{M}_{**}$ for $\beta < -1$. For $\mathrm{M}_\mathrm{i} > \mathrm{M}_*$ (or $\mathrm{M}_\mathrm{i} > \mathrm{M}_{**}$) there is failure of the "soliton" bunch. Therefore, further improvement of the plasma model is required. If the charge of the dust is negative, a soliton "bunch" does not form because for $\mathrm{M}_\mathrm{i} > \mathrm{M}_h$, the steadiness condition (see Sec. **2**) is violated and a shock wave does not form. We recall that for $\mathrm{M}_\mathrm{i} > \mathrm{M}_h$, $\nu_\mathrm{i} = \nu_\mathrm{d} = 0$, and $D_\mathrm{e} \neq 0$ in negatively charged dust, a rarefaction soliton forms: $n_\mathrm{d} < n_{\mathrm{d}0}$. The steadiness conditions are also violated for $\alpha < \alpha_{\min}$ or $\alpha > \alpha_{\max}$ [$\alpha_{\min}$ and $\alpha_{\max}$ are solutions of the equation $\mathrm{M}_h(\alpha, \beta) = \mathrm{M}_*(\alpha, \beta)$ for $\beta > -1$ or solutions of the equation $\mathrm{M}_h(\alpha, \beta) = \mathrm{M}_{**}(\alpha, \beta)$ for $\beta < -1$]. A special "resonant" situation arises for $\alpha = \alpha_{1,2}(\beta)$ (see Sec. **2**); as $\mathrm{M}_\mathrm{i} \to \mathrm{M}_h$, a region of elevated concentration of dust particles forms.

7. Let us obtain a criterion for the strong effect of charged components of a slightly ionized dust plasma on the neutral component. The solution of the closed problem at this stage involves considerable difficulties. Here we study a mechanism of this process — the interaction between charged particles and neutral particles using elastic collisions. We obtain a necessary condition for this interaction for the steady-state case, restricting ourselves to one-dimensional fields.

The processes in a slightly ionized nonisothermal dust plasma are described by the gas-dynamic equations (2) supplemented with the continuity equation and the equation of motion of the neutral component taking into account the effect of charged particles on neutral particles:

$$\begin{gathered}\frac{\partial n_\mathrm{n}}{\partial t} + \frac{\partial}{\partial x}(n_\mathrm{n}V_\mathrm{n}) = 0,\\ n_\mathrm{n}\Big(\frac{\partial}{\partial t} + V_\mathrm{n}\frac{\partial}{\partial x}\Big)V_\mathrm{n} = -a^2\frac{\partial n_\mathrm{n}}{\partial x} - \nu_{ni}n_\mathrm{n}(V_\mathrm{n} - V_\mathrm{i}) - \nu_{nd}n_\mathrm{n}(V_\mathrm{n} - V_\mathrm{d}),\\ \nu_{ni}n_\mathrm{n} = \nu_\mathrm{i}n_\mathrm{i}, \qquad \nu_{nd}n_\mathrm{n} = \nu_\mathrm{d}n_\mathrm{d}\end{gathered} \tag{11}$$

(a is the velocity of sound and V_n is an unknown function).

For the steady-state regime, we have the relation

$$\frac{d}{d\xi}[V_\mathrm{n} - a_0^2(c - V_\mathrm{n})^{-1} + n_{\mathrm{i}0}n_{\mathrm{n}0}^{-1}V_\mathrm{i} + n_{\mathrm{d}0}n_{\mathrm{n}0}^{-1}V_\mathrm{d}] - V_s\mathrm{M}_\mathrm{i}^{-1}n_{\mathrm{n}0}^{-1}(n_\mathrm{i} - \beta n_\mathrm{d})n_\mathrm{e}^{-1}\frac{dn_\mathrm{e}}{d\xi} = 0. \tag{12}$$

In the case of formation of regions with elevated concentration of charged particles, leading to the strong effect on the neutral component, the first two terms in relation (12) should be of the order of magnitude as its last term. Let us compare the orders of magnitude of the first and last terms in (12):

$$(n_\mathrm{e} - n_{\mathrm{e}0})n_{\mathrm{e}0}^{-1} \sim \delta_1^{-1}\delta_2\delta_3\delta_4,$$

$$\delta_1 = n_{\mathrm{e}0}n_{\mathrm{n}0}^{-1} \ll 1, \quad \delta_2 = n_\mathrm{e}|n_\mathrm{i} - \beta n_\mathrm{d}|^{-1}, \quad \delta_3 = aV_s^{-1} \approx (T_\mathrm{n}T_\mathrm{e}^{-1})^{1/2} < 1, \quad \delta_4 = V_\mathrm{n}a^{-1}.$$

In the absence of a substantially nonlinear process in the neutral component, the estimate $\delta_4 \leqslant 1$ is valid. The simplest situation with the formation of plasma bunches arises in the case of predominant effect of the dust component, where $\delta_2 \approx n_\mathrm{e}|\beta n_\mathrm{d}|^{-1} \ll 1$.

The strong effect of dust particles on the neutral component can be due to violation of the quasineutrality condition (1). In this case, the charged dust "scatter" compaction regions of neutral particles. However, a description of this process is beyond the model (1), (2), (11).

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